

Cosmological implications of quantum mechanics parametrization of dark energy

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Abstract. We consider the cosmology with the running dark energy. The parametrization of dark energy is derived from the quantum process of transition from the false vacuum state to the true vacuum state. This model is the generalized interacting Λ CDM model. We consider the energy density of dark energy parametrization, which is given by the Breit-Wigner energy distribution function. The idea of the process of the quantum mechanical decay of unstable states was formulated by Krauss and Dent. We used this idea in our considerations. In this model is an energy transfer in the dark sector. In this evolutionary scenario the universe starts from the false vacuum state and goes to the true vacuum state of the present day universe. The intermediate regime during the passage from false to true vacuum states takes place. In this way the cosmological constant problem can be tried to solve. We estimate the cosmological parameters for this model. This model is in a good agreement with the astronomical data and is practically indistinguishable from Λ CDM model.

1. Introduction

We look for an alternative cosmological model, for the Λ CDM model, because we want to find a solution of the cosmological constant problem. Our proposition of the solution of the problem of the cosmological constant parameter assumes that the vacuum energy is given by the fundamental theory [1]. We apply quantum mechanics as a fundamental theory, which determines cosmological parameters and explain how cosmological parameters change during the cosmic evolution. The discussion about the cosmological constant problem can be found in papers [1]–[14].

Coleman *et al.* [15]–[17] showed that even if the early Universe is too cold to activate the energy transition to the minimum energy state then a quantum decay, from the false vacuum to the true vacuum, is still possible through a barrier penetration (by quantum tunneling). The discovery of the Higgs-like resonance at 125-126 GeV [18]–[21] gave the discussion about the instability of the false vacuum. In the neighborhood of the Planck epoch the Higgs mass $m_h < 126\text{GeV}$ causes that the electroweak vacuum is in a metastable state [19]. So the cosmological models of the early time Universe should respect the instability of the Higgs



vacuum. The properties of the quantum mechanical decay process of metastable states should help to understand the properties of the observed universe[22] because the decay of the false vacuum is the quantum process [15, 16, 17].

We assume the Breit-Wigner energy distribution function as a model of the process of the energy transition from the false vacuum to the true vacuum. In consequence the parametrization of the dark energy has the following form

$$\rho_{de} = E_0 + E_R \frac{\alpha}{1-\alpha} \Re \left(\frac{J(t)}{I(t)} \right) \text{ or } \rho_{de} = \Lambda_{\text{bare}} + E_R \left[1 + \frac{\alpha}{1-\alpha} \Re \left(\frac{J(t)}{I(t)} \right) \right], \quad (1)$$

where α , E_R are model parameters, which describe the variation from Λ CDM model and $\Lambda_{\text{bare}} = E_0 - E_R$. The values of α parameter belong to interval $(0, 1)$. When the α parameter or E_R is equal zero than this model is equivalent to the standard cosmological model. We assume that $8\pi G = c = 1$.

The functions $J(t)$ and $I(t)$ are described by the following formulas

$$J(t) = \int_{-\frac{1-\alpha}{\alpha}}^{\infty} \frac{\eta}{\eta^2 + \frac{1}{4}} e^{-i\eta\tau} d\eta \text{ and } I(t) = \int_{-\frac{1-\alpha}{\alpha}}^{\infty} \frac{1}{\eta^2 + \frac{1}{4}} e^{-i\eta\tau} d\eta. \quad (2)$$

The exact solutions of integrals $J(t)$ and $I(t)$ are given by

$$J(\tau) = \frac{1}{2} e^{-\tau/2} \left(-2i\pi + e^{\tau} E_1 \left(\left[\frac{1}{2} - \frac{i(1-\alpha)}{\alpha} \right] \tau \right) + E_1 \left(\left[-\frac{1}{2} - \frac{i(1-\alpha)}{\alpha} \right] \tau \right) \right), \quad (3)$$

$$I(\tau) = 2\pi e^{-\tau/2} \left(1 + \frac{i}{2\pi} \left(-e^{\tau} E_1 \left(\left[\frac{1}{2} - \frac{i(1-\alpha)}{\alpha} \right] \tau \right) + E_1 \left(\left[-\frac{1}{2} - \frac{i(1-\alpha)}{\alpha} \right] \tau \right) \right) \right), \quad (4)$$

where $\tau = \frac{\alpha(E_0 - \Lambda_{\text{bare}})}{\hbar(1-\alpha)} V_0 t$ and V_0 is the volume of the Universe in the Planck epoch. We assume that $V_0 = 1$. The function $E_1(z)$ is the exponential integral and is given by the formula: $E_1(z) = \int_z^{\infty} \frac{e^{-x}}{x} dx$ (see [23]).

2. Preliminaries: unstable states

The parametrization of the dark energy transition from the false vacuum state to the true vacuum state following from the quantum properties of a such process will be used. This process is a quantum decay process. Quantum unstable systems are characterized by their survival probability. So if the system is in the initial unstable state $|\phi\rangle \in \mathcal{H}$, (\mathcal{H} is the Hilbert space of states of the considered system), which was prepared at the initial instant $t_0 = 0$, then its survival probability (the decay law), $\mathcal{P}(t)$, of the unstable state $|\phi\rangle$ decaying in vacuum equals $\mathcal{P}(t) = |A(t)|^2$, where $A(t)$ is the probability amplitude of finding the system at the time t in the initial unstable state $|\phi\rangle$, $A(t) = \langle\phi|\phi(t)\rangle$, and $|\phi(t)\rangle$ is the solution of the Schrödinger equation for the initial condition $|\phi(0)\rangle = |\phi\rangle$, which has the following form

$$i\hbar \frac{\partial}{\partial t} |\phi(t)\rangle = \mathfrak{H} |\phi(t)\rangle. \quad (5)$$

Here $|\phi\rangle, |\phi(t)\rangle \in \mathcal{H}$, and \mathfrak{H} is the total self-adjoint Hamiltonian for the system considered. The spectrum, $\sigma_c(\mathfrak{H})$, of \mathfrak{H} is assumed to be bounded from below: $\sigma_c(\mathfrak{H}) = [E_{\min}, +\infty)$ of \mathfrak{H}). By using the basis in \mathcal{H} formed by normalized eigenvectors $|E\rangle$, $E \in \sigma_c(\mathfrak{H})$ of \mathfrak{H} and expanding $|\phi\rangle$ in terms of these eigenvectors one can express the amplitude $A(t)$ as the following Fourier integral

$$A(t) \equiv \int_{E_{\min}}^{\infty} \omega(E) e^{-\frac{i}{\hbar} E t} dE, \quad (6)$$

where $\omega(E) > 0$ (see: [24]). Hence the amplitude $A(t)$, and thus the decay law $\mathcal{P}(t)$ of the unstable state $|\phi\rangle$, are completely determined by the density of the energy distribution $\omega(E)$ for the system in this state [24]. The amplitude $A(t)$ contains information about the decay law $\mathcal{P}_\phi(t)$ of the state $|\phi\rangle$: About the decay rate γ_ϕ^0 of this state, as well as the energy \mathcal{E}_ϕ^0 of the system in this state. This information can be extracted from $A(t)$. It can be done by using the rigorous equation governing the time evolution in the subspace of unstable states, $\mathcal{H}_\parallel \ni |\phi\rangle_\parallel \equiv |\phi\rangle$. Such an equation follows from Schrödinger equation (5) for the total state space \mathcal{H} .

By using the Schrödinger equation (5) one can find that within the problem considered $i\hbar \frac{\partial}{\partial t} \langle \phi | \phi(t) \rangle = \langle \phi | \mathfrak{H} | \phi(t) \rangle$. From this relation a conclusion follows that the amplitude $A(t)$ satisfies the following equation

$$i\hbar \frac{\partial A(t)}{\partial t} = h(t) A(t), \quad (7)$$

where

$$h(t) = \frac{\langle \phi | \mathfrak{H} | \phi(t) \rangle}{A(t)} \equiv \frac{i\hbar}{A(t)} \frac{\partial A(t)}{\partial t}, \quad (8)$$

and $h(t)$ is the effective Hamiltonian governing the time evolution in the subspace of unstable states $\mathcal{H}_\parallel = \mathbb{P}\mathcal{H}$, where $\mathbb{P} = |\phi\rangle\langle\phi|$ (see [26] and also [27, 28, 29] and references therein). The subspace $\mathcal{H} \ominus \mathcal{H}_\parallel = \mathcal{H}_\perp \equiv \mathbb{Q}\mathcal{H}$ is the subspace of decay products. Here $\mathbb{Q} = \mathbb{I} - \mathbb{P}$. In general $h(t)$ is a complex function of time and in the case of \mathcal{H}_\parallel of dimension two or more the effective Hamiltonian governing the time evolution in such a subspace it is a non-hermitian matrix H_\parallel or non-hermitian operator. There is $h(t) = E_\phi(t) - \frac{i}{2}\Gamma_\phi(t)$, and $E_\phi(t) = \Re[h(t)]$, $\Gamma_\phi(t) = -2\Im[h(t)]$, are the instantaneous energy (mass) $E_\phi(t)$ and the instantaneous decay rate, $\Gamma_\phi(t)$ [26, 27, 28]. It is convenient to use relations (7), (8) when the density $\omega(E)$ is given and one wants to find the instantaneous energy $E_\phi(t)$ and decay rate $\Gamma_\phi(t)$: Inserting $\omega(E)$ into (6) one obtains the amplitude $A(t)$ and then using (8) one finds the $h(t)$ and thus $E_\phi(t)$ and $\Gamma_\phi(t)$. The simplest choice is to take $\omega(E)$ having the Breit-Wigner form: $\omega(E) \equiv \omega_{BW}(E) \stackrel{\text{def}}{=} \frac{N}{2\pi} \Theta(E - E_{\min}) \frac{\Gamma_0}{(E - E_0)^2 + (\frac{\Gamma_0}{2})^2}$, where N is a normalization constant and $\Theta(E) = 1$ for $E \geq 0$ and $\Theta(E) = 0$ for $E < 0$. The parameters E_0 and Γ_0 correspond with the energy of the system in the unstable state and its decay rate at the exponential (or canonical) regime of the decay process. Inserting $\omega_{BW}(E)$ into formula (6) for the amplitude $A(t)$ after some algebra one finds that

$$A(t) = \frac{N}{2\pi} e^{-\frac{i}{\hbar} E_0 t} I_\beta \left(\frac{\Gamma_0 t}{\hbar} \right), \quad (9)$$

where $I_\beta(\tau)$ is the integral given by (4) with $\beta = (1 - \alpha)/\alpha \equiv (E_0 - E_{\min})/\Gamma_0$. Here $\tau = \frac{\Gamma_0 t}{\hbar} \equiv \frac{t}{\tau_0}$, τ_0 is the lifetime. An explicit form of $I_\beta(t)$ is given by the formula (4). Next using this $A(t)$ given by the relation (9), and the relation (8) defining the effective Hamiltonian $h_\phi(t)$ one finds that within the Breit-Wigner model considered

$$h(t) = i\hbar \frac{1}{A(t)} \frac{\partial A(t)}{\partial t} = E_0 + \Gamma_0 \frac{J_\beta(\frac{\Gamma_0 t}{\hbar})}{I_\beta(\frac{\Gamma_0 t}{\hbar})}, \quad (10)$$

where $J_\beta(\tau)$ is given by the formula (3) for $\beta = (1 - \alpha)/\alpha$. It is important to be aware of the following problem: Namely from the definition of $J_\beta(\tau)$ one can conclude that $J_\beta(0)$ is undefined ($\lim_{\tau \rightarrow 0} J_\beta(\tau) = \infty$). This is because within the model defined by the Breit-Wigner distribution of the energy density, $\omega_{BW}(E)$, the expectation value of \mathfrak{H} , that is $\langle \phi | \mathfrak{H} | \phi \rangle$ is not finite. So all the consideration based on the use of $J_\beta(\tau)$ are valid only for $\tau > 0$. Note that simply $J_\beta(\tau) \equiv i \frac{\partial I_\beta(\tau)}{\partial \tau}$, which allows one to find analytical form of $J_\beta(\tau)$ having such a form for $I_\beta(\tau)$.

We need to know the energy $E_\phi(t) = \Re[h(t)]$ of the system in the unstable state $|\phi\rangle$ considered. Within the Breit-Wigner model one finds that

$$E_\phi(t) = E_0 + \Gamma_0 \Re \left[\frac{J_\beta(\frac{\Gamma_0 t}{\hbar})}{I_\beta(\frac{\Gamma_0 t}{\hbar})} \right]. \quad (11)$$

There are cosmological scenarios that predict the possibility of decay of the Standard Model vacuum at an inflationary stage of the evolution of the universe (see eg. [31] and also [32] and reference therein) or earlier. Such scenario correspond with the hypothesis analyzed by Krauss and Dent [22]: The hypothesis that some false vacuum regions do survive well up to the cross-over time T or later was considered therein, (where T is the time when exponential and late time inverse power law contributions to the survival amplitude begin to be comparable). The fact that the decay of the false vacuum is the quantum decay process means that state vector corresponding to the false vacuum is a quantum unstable (or metastable) state. Therefore all the general properties of quantum unstable systems must also occur in the case of such a quantum unstable state as the false vacuum. This applies in particular to such properties as late time deviations from the exponential decay law and properties of the energy $E_0^{false}(t)$ of the system in the quantum false vacuum state at late times $t > T$: In [33] it was pointed out the energy of those false vacuum regions which survived up to T and much later differs from E_0^{false} [33]. Simply within the cosmological scenario in which the decay of false vacuum is assumed the unstable state $|\phi\rangle$ corresponds with the false vacuum state: $|\phi\rangle = |0\rangle^{false}$. Then $|0\rangle^{true}$ is the true vacuum state corresponding to the true minimal energy. In such a case $E_0 = E_0^{false}$ is the energy of a state corresponding to the false vacuum measured at the canonical decay time (the exponential decay regime) and E_0^{true} is the energy of true vacuum (i.e., the true ground state of the system), so $E_0^{true} \equiv E_{min}$. If one wants to generalize the above considerations to quantum field theory one should take into account among others a volume factor so that survival probabilities per unit volume should be considered and similarly the energies and the decay rate: $E \mapsto \rho(E) = \frac{E}{V_0}$, $\Gamma_0 \mapsto \gamma = \frac{\Gamma_0}{V_0}$, where $V_0 = V(t_0)$ is the volume of the considered system at the initial instant t_0 , when the time evolution starts. It is easy to see that the mentioned changes $E \mapsto \frac{E}{V_0}$ and $\Gamma_0 \mapsto \frac{\Gamma_0}{V_0}$ do not change integrals $I_\beta(t)$ and $J_\beta(t)$. Similarly in such a situation the parameter $\beta = \frac{E_0 - E_{min}}{\Gamma_0}$ does not change. This means that the relation (11) can be replaced by corresponding relations for the densities ρ_{de} or Λ (see, eg., [29, 35, 36]).

3. Cosmological equations with $\rho_{de} = \Lambda_{bare} + E_R \left[1 + \frac{\alpha}{1-\alpha} \Re \left(\frac{J(t)}{I(t)} \right) \right]$

In this paper, our model is introduced as the covariant theory from the following action

$$S = \int \sqrt{-g} (R + \mathcal{L}_m) d^4x, \quad (12)$$

where R is the Ricci scalar, \mathcal{L}_m is the Lagrangian for the barotropic fluid ($\mathcal{L}_m = -\rho_{tot} \left(1 + \int \frac{p_{tot}(\rho_{tot})}{\rho_{tot}^2} d\rho_{tot} \right)$, where ρ_{tot} is the total density of fluid and $p_{tot}(\rho_{tot})$ is the total pressure of fluid [37]) and $g_{\mu\nu}$ is the metric tensor. Here, the signature of the metric tensor is assumed as $(+, -, -, -)$. We consider the flat model (the constant curvature is zero).

We assume that the fluid consists of the baryonic matter ρ_b , the dark matter ρ_{dm} and the dark energy ρ_{de} . We treat the baryonic matter and the dark matter like dust ($p_b(\rho_b) = 0$ and $p_{dm}(\rho_{dm}) = 0$) and the equation of state for the dark energy is given by $p_{de}(\rho_{de}) = -\rho_{de}$.

The Friedmann and acceleration equations can be found by variation action (12) by the metric $g_{\mu\nu}$. Then we get:

$$3H^2 = 3\frac{\dot{a}^2}{a} = \rho_{tot} = \rho_b + \rho_{dm} + \rho_{de} \text{ and } \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_{tot} + 3p_{tot}(\rho_{tot})) = \rho_b + \rho_{dm} - 2\rho_{de}, \quad (13)$$

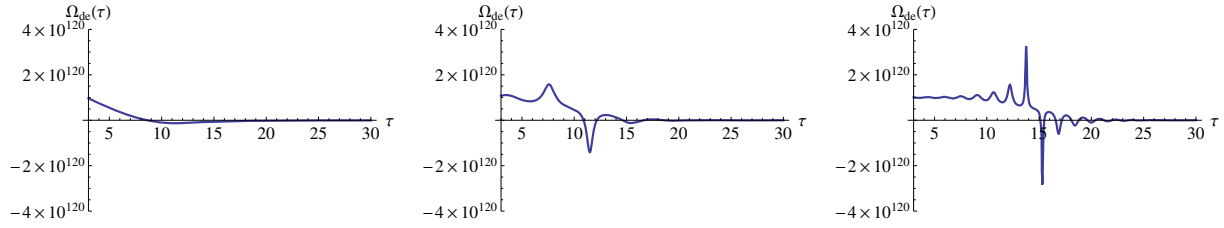


Figure 1. The dependence $\Omega_{\text{de}}(\tau)$ for $\alpha = 0.2$ (right figure), $\alpha = 0.4$ (medium figure), $\alpha = 0.8$ (left figure) and $\frac{E_0}{3H_0^2} = 10^{120}$. The time τ for the left figure is given in unit $5.3 \times 10^{-145} \text{ s}$, for the center figure is given in unit $2.0 \times 10^{-145} \text{ s}$ and for the right figure is given in unit $3.3 \times 10^{-146} \text{ s}$.

where $H = \frac{\dot{a}}{a}$ is the Hubble function,

From the above equations we can obtain the conservation equation

$$\dot{\rho}_{\text{tot}} = -3H(\rho_{\text{tot}} + p_{\text{tot}}(\rho_{\text{tot}})) \text{ or in the equivalent form } \dot{\rho}_{\text{m}} = -3H\rho_{\text{m}} - \dot{\rho}_{\text{de}}, \quad (14)$$

where $\rho_{\text{m}} = \rho_{\text{b}} + \rho_{\text{dm}}$.

If we introduce the interaction Q in the dark sector then Eq. (14) can be rewritten as

$$\dot{\rho}_{\text{b}} = -3H\rho_{\text{b}}, \quad \dot{\rho}_{\text{dm}} = -3H\rho_{\text{dm}} + Q \text{ and } \dot{\rho}_{\text{de}} = -Q. \quad (15)$$

If $Q > 0$ then the energy flows from the dark energy to the dark matter. If $Q < 0$ then the energy flows from the dark matter to the dark energy.

Fig. 1 shows the diagrams of the evolution $\Omega_{\text{de}}(\tau) = \frac{\rho_{\text{de}}(\tau)}{3H^2}$, where H_0 is the present value of the Hubble function, with respect of the time τ for different values of model parameter α ($\alpha = 0.2, 0.4, 0.8$) and $\frac{E_0}{3H_0^2} = 10^{120}$. We can see that the oscillations disappear for $\alpha > 0.4$ and the number of oscillations as well as the length of this intermediate phase decreases as the parameter α grows. On this figure we can see that the beginning value of the dark energy density, which is equal $\Omega_{\text{de}} \approx 10^{120}$, is reduced to the present value of the dark energy density, which is $\Omega_{\text{de}} \approx 0.7$. This final value of Ω_{de} does not depend on the values of model parameters α and $\frac{E_0}{3H_0^2}$. This mechanism can be a trying to solve the cosmological constant problem. The characteristic of the intermediate oscillatory phase is depended on the parameter α .

4. Statistical analysis

In statistical analysis, we use the following astronomical data: supernovae of type Ia (SNIa) (Union 2.1 dataset [39]), BAO data (Sloan Digital Sky Survey Release 7 (SDSS DR7) dataset at $z = 0.275$ [40], 6dF Galaxy Redshift Survey measurements at redshift $z = 0.1$ [41], and WiggleZ measurements at redshift $z = 0.44, 0.60, 0.73$ [43]), measurements of the Hubble parameter $H(z)$ of galaxies [55, 56, 57], the Alcock-Paczynski test (AP) [45, 46] (data from [47, 48, 49, 50, 51, 52, 42, 53, 54].) and measurements of CMB [58] and lensing by Planck, and low- ℓ polarization from the WMAP (WP).

We take the formula for likelihood function as $L_{\text{tot}} = L_{\text{SNIa}} L_{\text{BAO}} L_{\text{AP}} L_{H(z)} L_{\text{CMB+lensing}}$. The likelihood function $L_{\text{SNIa}} = \text{Exp} \left[-\frac{1}{2} [A - B^2/C + \log(C/(2\pi))] \right]$ is for SNIa, where $A = (\mu^{\text{obs}} - \mu^{\text{th}}) \mathbb{C}^{-1} (\mu^{\text{obs}} - \mu^{\text{th}})$, $B = \mathbb{C}^{-1} (\mu^{\text{obs}} - \mu^{\text{th}})$, $C = \text{Tr} \mathbb{C}^{-1}$ and \mathbb{C} is a covariance matrix for SNIa, μ^{obs} is the observer distance modulus and μ^{th} is the theoretical distance modulus. The likelihood function $L_{\text{BAO}} = \text{Exp} \left[-\frac{1}{2} \left(\mathbf{d}^{\text{obs}} - \frac{r_s(z_d)}{D_V(\mathbf{z})} \right) \mathbb{C}^{-1} \left(\mathbf{d}^{\text{obs}} - \frac{r_s(z_d)}{D_V(\mathbf{z})} \right) \right]$ is for BAO, where $r_s(z_d)$ is the sound horizon at the drag epoch [59, 44]. The likelihood function

Table 1. The best fit and errors for the estimated model with H_0 from the interval (66.0, 72.0), $\Omega_{m,0}$ from the interval (0.27, 0.34). $\Omega_{b,0}$ is assumed as 0.048468.

parameter	best fit	68% CL	95% CL
H_0	68.82 km/(s Mpc)	+0.61 −0.55	+0.98 −0.92
$\Omega_{m,0}$	0.3009	+0.0079 −0.0084	+0.0133 −0.0134

$L_{H(z)} = \text{Exp} \left[-\frac{1}{2} \sum_{i=1}^N \left(\frac{H(z_i)^{\text{obs}} - H(z_i)^{\text{th}}}{\sigma_i} \right)^2 \right]$ is for measurements of the Hubble parameter $H(z)$ of galaxies. The likelihood function $L_{AP(z)} = \text{Exp} \left[-\frac{1}{2} \sum_{i=1}^N \left(\frac{AP(z_i)^{\text{obs}} - AP(z_i)^{\text{th}}}{\sigma_i} \right)^2 \right]$ is for AP, where $AP(z)^{\text{th}} \equiv \frac{H(z)}{z} \int_0^z \frac{dz'}{H(z')}$ and $AP(z_i)^{\text{obs}}$ are observational data. The likelihood function $L_{\text{CMB+lensing}} = \text{Exp} \left[-\frac{1}{2} (\mathbf{x}^{\text{th}} - \mathbf{x}^{\text{obs}}) \mathbb{C}^{-1} (\mathbf{x}^{\text{th}} - \mathbf{x}^{\text{obs}}) \right]$ is for CMB and lensing, where \mathbb{C} is the covariance matrix with the errors, \mathbf{x} is a vector of the acoustic scale l_A , the shift parameter R and $\Omega_b h^2$ where $l_A = \frac{\pi}{r_s(z^*)} c \int_0^{z^*} \frac{dz'}{H(z')}$ and $R = \sqrt{\Omega_{m,0} H_0^2} \int_0^{z^*} \frac{dz'}{H(z')}$, where z^* is the redshift of the epoch of the recombination [59].

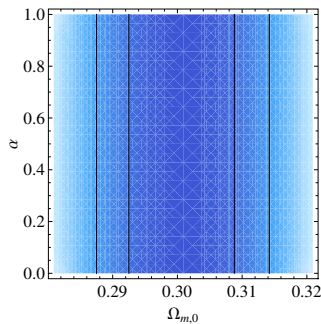


Figure 2. The diagram presents the intersection of the likelihood function of two model parameters ($\Omega_{m,0}$, α), with the marked 68% and 95% confidence levels. The plane of the intersection is the best fit of H_0 and $E_0/(3H_0^2) = 10^{120}$.

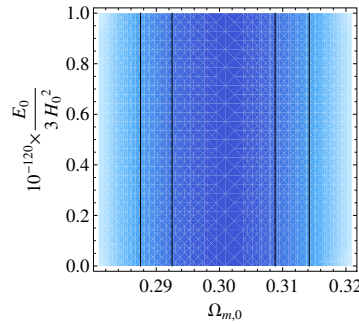


Figure 3. The diagram presents the intersection of the likelihood function of two model parameters ($\Omega_{m,0}$, $\frac{E_0}{3H_0^2}$), with the marked 68% and 95% confidence levels. The plane of the intersection is the best fit of H_0 and $\alpha = 0.1$.

We used our own code CosmoDarkBox in estimation of model parameter. This code uses the Metropolis-Hastings algorithm [60, 61].

In statistical analysis, we estimated four model parameters: H_0 , $\Omega_{m,0} = \frac{\rho_{m,0}}{3H_0^2}$, α and E_0 . Our statistical results are completed in Table 1. Figs. 2-3 show intersections of the likelihood function with 68% and 95% confidence level projections.

Changing of the values of α and E_0 parameter has not an influence for the values of the likelihood function. The changing of the values of the likelihood function are beyond abilities of numerical methods. We interpret this as the lack of sensitive of the present evolution of the universe for changing of α and E_0 parameter.

5. Conclusion

The main goal of our paper was to analyze the cosmological model with the running dark energy as well as matter in the dust form. We discussed the evolution of the dark energy with the respect that the decay of false vacuum to the true vacuum is a quantum decay process. In this case the cosmological model includes the interaction in the dark sector.

We found the intermediate phase of oscillations, in the early universe, between phases of the constant dark energy. There is a mechanism of jumping of the value of energy density of dark energy from the initial value of $E_0 = 10^{120}$ to present value of the cosmological constant of 0.7.

In this intermediate phase the amplitude of oscillatory increases, then there is a jump down followed by the decreasing oscillations. This type of oscillation appears for $0 < \alpha < 0.4$, where α is a model parameter. The number of oscillations as well as the length of this intermediate phase decreases as the parameter α grows. For $\alpha > 0.4$ the oscillations, in the intermediate phase, disappear and only the jump down of energy density of dark energy remains. The jump down mechanism can lead to solve the cosmological constant problem.

In this model, the energy transfer, in the early universe in the dark sector, is negligible because the energy density of dark energy is significantly lower than the energy density of dark matter.

From the statistical analysis, we found that the present value of the energy density of the dark energy is independent of the assumed values of the parameters α and E_0 . The Λ CDM model is an attractor for the all models with different values of parameters α and E_0 . The final interval of evolution for which we have data at dispose is identical for whole class of models. So it is impossible to find the best fit values of model parameters (degeneration problem).

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